THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 5232 Fractal Geometry

TIME: 3 hrs **Max.Marks: 75**

Answer any FIVE questions $\mathbf{5 \times 15 = 75}$

1. a)Define Hausdorff distance and prove that $h(A\cup B, C\cup D)\le h(A,C)\vee h(B,D)$
   b)State and prove the extension lemma
2. Prove that the space of fractals on a complete metric space is complete and also prove that if $\{A_n \in H(X)\}_{n=1}^{\infty}$ is a Cauchy sequence then $\underset{n\to\infty}{lt} A_n = A$ and
   $A = \{x \in X \,/\, \ni a\, cauchy\; sequence \{x_n \in A_n\}_{n=1}^{\infty}\; converges\; to\; x\}$
3. a) State and prove the contraction mapping theorem b) Find the affine transformation in R^2that takes the triangle with vertices at (0,0), (0,1)& (1,0) is mapped onto the triangle with vertices at (7, 7), (4, 8)&(4, 9)
4. a) Prove that code space of two or ,ore symbols is uncountable
   b)Find the contractivity factor of the IFS given below
   $\left\{R^2 / w_1(x_1,x_2) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2}\end{pmatrix}\begin{pmatrix} x_1 \\ x_2\end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2}\end{pmatrix} \;\&\; w_2(x_1,x_2) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2}\end{pmatrix}\begin{pmatrix} x_1 \\ x_2\end{pmatrix}\right\}$ and prove that
   $A = \{(x,y)/x = y,\; x,y \in [01]\}$ is its attractor and also represent it geometrically

5. Let (X, d) be a complete metric space let $\{X : \omega_n,\, n = 1,2,....N\}$ be the hyperbolic IFS. Let 'A' denote the attractor of the IFS. Let (Σ, d_c) denote the code space associated with the IFS. $\forall \sigma \in \Sigma,\, n \in N \,\&\, x \in X$ $\phi(\sigma,n,x) = \omega_{\sigma_1}\circ\omega_{\sigma_2}\circ\omega_{\sigma_3}\circ............\omega_{\sigma_n}(x)$ then prove that $\phi(\sigma) = \underset{n\to\infty}{lt}\phi(\sigma,n,x)$ exists belongs to 'A' and is independent of x in X. If 'K' is a compact subset of X then prove that the convergence is uniform over x in K and prove that the function $\phi : \Sigma \to A$ is continuous and onto.
6. State and prove the Box Counting Theorem
7. Let $(X_1,d_1),(X_2,d_2)$ be metrically equivalent metric spaces and let $\theta : X_1 \to X_2$ be a transformation that provides equivalence of the spaces. If $A_1 \in H(X_1)$ has fractal dimension D, then prove that $A_2 = \theta(A_1)$ has the same fractal dimension D