PGM 4435   **TIME:3HRS**

GRAPH THEORY   **MARK:75**

Answer any **FIVE** questions   5x15=75

1. Prove that a graph G is bipartite if and only if it contains no odd cycle.

2. State Kruskal's algorithm to find a minimum weight spanning tree for a connected weighted graph. Also prove that the spanning tree constructed using Kruskal's algorithm is a minimum weight spanning tree.

3. Prove that a connected graph has an euler trail if and only if it has at most two vertices of odd degree.

4. Let G be a graph and let u and v be non adjacent vertices in G such that $d_G(u) + d_G(v) \geq p$. Prove that G is Hamiltonian if and only if G +uv is Hamiltonian.

5. State and prove Tutte's theorem

6. Prove that a graph with $v \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally-disjoint paths.

7. Prove that for any graph $G, \ \chi(G) \leq \Delta(G) + 1$.