THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT/MAS 2442 / 2512 **TIME: 3 HRS**

LINEAR ALGEBRA AND LATTICES /ALGEBRA III **MAX: 75**

ANSWER ANY FIVEQUESTION: **5× 15 = 75**

1. Let V be the vector space of all functions from $R \to R$. Let $V_e = \{f \in V / f \ is \ even\}$, $V_0 = \{f \in V / f \ is \ odd\}$. Prove that V_e and V_0 are subspaces of V and $V = V_e \oplus V_0$..

2. Let V be a vector space over F and W a subspace of V. Let $V/W = \{W + v/v \in V\}$. Prove that V/W is a vector space over F under the following operations.
(i) $(W + v_1) + (W + v_2) = W + v_1 + v_2$
(ii) $\alpha(W + v_1) = W + \alpha v_1$.

3. Let V be a vector space over a field F. Let $S,T \subseteq V$ then prove that the following:
(i) $S \subseteq T \Longrightarrow L(S) \subseteq L(T)$
(ii) $L(S \cup T) = L(S) + L(T)$
(iii) $L(S) = S$ iff S is a subspace of V

4. Let V be a finite dimensional vector space over a field F. Let A and B be subspace of V. Then prove that $\dim(A + B) = \dim A + \dim B - \dim(A \cap B)$.

5. State and prove the fundamental theorem of homomorphism for groups.

6. State and prove Schwartz's and triangle inequality

7. Prove that every finite dimensional inner product space has an orthonormal basis.

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