



# THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2<sup>nd</sup> Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

## Backlog Arrear Examination, March 2021

**MAT 1405/1444**

**Max.Marks:75**

**Geometry**

**Duration:3hours**

**Answer any FIVE Questions:**

**5×15=75**

1. Show that the origin lies in the acute angle between the planes  
 $x + 2y + 2z = 9$ ,  $4x - 3y + 12z + 13 = 0$ . Find the planes bisecting the angles between them and point out which bisects the obtuse angle.
2. Show that the straight lines whose direction cosines are given by  $al + bm + cn = 0$ ,  
 $fmn + gnl + hlm = 0$  are perpendicular if  $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$  and parallel if  $\sqrt{af} + \sqrt{hg} + \sqrt{ch} = 0$
3. Find the equations of the image of the line  $\frac{x-1}{2} = \frac{y+2}{-5} = \frac{z-3}{2}$  in the plane  
 $2x - 3y + 2z + 3 = 0$ .
4. (i) Prove that the lines  $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$ ;  $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$  are coplanar. Find also their point of intersection and the plane through them.  
(ii) The lengths of two opposite edges of a tetrahedron are  $a, b$ ; their shortest distance is equal to  $d$  and the angle between them  $\theta$ . Prove that the volume is  $\frac{abd \sin \theta}{6}$ .
5. Find the equation of the sphere which passes through the circle  
 $x^2 + y^2 + z^2 - 2x - 4y = 0$ ,  $x + 2y + 3z = 8$  and touches the plane  $4x + 3y = 25$ .
6. (i) Find the equation of the sphere through the four points  $(2,3,1)$ ,  $(5, -1, 2)$ ,  $(4,3,-1)$  and  $(2,5,3)$ .  
(ii) Show that the plane  $2x - y - 2z = 16$  touches the sphere  $x^2 + y^2 + z^2 - 4x + 2y + 2z - 3 = 0$  and find the point of contact.
7. (i) State and Prove Frenet-Serret Formulae.  
(ii) Prove that the locus of the centre of curvature is an evolute only when the curve is plane.