THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 5332 **MAX: 75 marks**

FUZZY MATHEMATICS **TIME: 3 hrs**

Answer any five questions: $5 \times 15 = 75$ marks

1. a) Let A, B$\in \mathcal{F}(X)$. Then prove that the following properties hold for all $\alpha, \beta \in [0,1]$:

   i)$\alpha_{(A\cap B)} = \alpha_A \cap \alpha_B$ ii)$\alpha_{\bar{A}} = (1-\alpha)+_{\bar{A}}$

   iii) $\alpha +_{(A\cup B)} = \alpha +_A \cup \alpha +_B$ iv)$A \subseteq B \iff \alpha_A \subseteq \alpha_B$.

   b) State and prove the second decomposition theorem.

2. a) Let $f: X \to Y$ be an arbitrary crisp function. Then prove that for any $A_i \in \mathcal{F}(X)$ and any $B_i \in \mathcal{F}(Y), i \in I$, the following properties hold:

   i)$f\left(\bigcup_{i\in I} A_i\right) = \bigcup_{i\in I} f(A_i)$ ii)if $B_1 \subseteq B_2$, then $f^{-1}(B_1) \subseteq f^{-1}(B_2)$

   iii)$f^{-1}(\bigcap_{i\in I} B_i) = \bigcap_{i\in I} f^{-1}(B_i)$ iv) $\overline{f^{-1}(B)} = f^{-1}(\bar{B})$

   b) Give an example to show that the set inclusion in $A \subseteq f^{-1}(f(A))$ cannot be replaced with the equality.

3. State and prove the first characterization theorem of fuzzy complements.

4. State and prove the characterization theorem for fuzzy numbers.

5. With usual notations, prove the following

   i)MIN(A,B)=MIN(B,A)

   ii)MIN[MIN(A,B),C]=MIN[A,MIN(B,C)]

   iii)MIN(A,MAX(A,B))=A

   iv)MIN(A,MAX(B,C))=MAX[MIN(A,B),MIN(A,C)]

6. Suppose $A(x) = \begin{cases} 0 \; for\; x \leq -1 \; and \; x > 3 \\ \frac{x+1}{2} \; for -1 < x \leq 1 \\ \frac{3-x}{2} \; for\; 1 < x \leq 3 \end{cases}$ and $B(x) = \begin{cases} 0 \; for\; x \leq 1 \; and \; x > 5 \\ \frac{x-1}{2} \; for\; 1 < x \leq 3 \\ \frac{5-x}{2} \; for\; 3 < x \leq 5 \end{cases}$

   Find $A + B, A.B$ and A/B.

7. a) Write down the procedure to solve the fuzzy relational equation $p \circ Q = r$ where $\circ$ is the max-min composition.

b) Solve the following fuzzy relation equation for the max-min composition

$$P \circ \begin{pmatrix} .5 & 0 & .3 & 0 \\ .4 & 1 & .3 & 0 \\ 0 & .1 & 1 & .1 \\ .4 & .3 & .3 & .5 \end{pmatrix} = [.5\ \ .3\ \ .3\ \ .1]\ .$$