MAT/ MAS 1603   **MARKS: 75**

ALGEBRA   **TIME: 3HRS**

ANSWER ANY FIVE QUESTIONS: **5×15=75**

1. Show that $\log \sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \ldots$

2. Sum the series to n terms

$$\frac{8}{1.2.3}\left(\frac{5}{7}\right) + \frac{9}{2.3.4}\left(\frac{5}{7}\right)^2 + \frac{10}{3.4.5}\left(\frac{5}{7}\right)^3 + \ldots$$

3. If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two, prove that $p^3 + 8r = 4pq$.

4. Solve the equation $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$ by finding the rational roots.

5. (a) Find the maximum value of $(3 - x)^5(2 + x)^4$ when x lies between 3 and -2.
   (b)Find the remainder obtained in dividing 2^{46} by 47.

6. State and prove Fermat's theorem.

7. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

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