THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 5546/5436 **Time: 3 Hrs**
Statistics – II / Statistical Inference and Stochastic processes **Marks: 75**

Answer any **FIVE** questions. **5 x 15 = 75**

1) State Likelihood Test, Wald Type Test, Score Test. Also apply the above tests for theLaplace Location Model.

2) State and prove Rao-Cramer Lower bound. Also derive the information matrix.

3) Write down the six Regularity conditions. Also assume $X_1, \ldots, X_n$ are iid with pdf $f(x;\theta_0)\ for \theta_0 \in \Omega$ such that the regularity conditions (R0) to (R5) are satisfied. Suppose further that the Fisher information satisfies $0 < I(\theta_0) < \infty$. Then prove any consistent sequence of solutions of the mle equations satisfies $\sqrt{n}(\hat{\theta}_n - \theta_0)$ converges in distribution to $N\left(0, 1/I(\theta)\right)$.

4) Define best critical region. Also State and prove Neyman-Pearson theorem.

5) (a) Define sufficient statistic for a parameter θ.
(b) State and prove Neyman (Factorization) theorem

6) Explain Polya's urn model in detail.

7) (a) Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $\{X_n, n \geq 1\}$ be the digit leaving nth stage of system and X_0 be the digit entering the first stage (leaving the 0th stage). At each stage there is a constant probability q that the digit which enters will be transmitted unchanged (the digit will remain unchanged when it leaves), and probability p otherwise (the digit changes when it leaves) p+q=1. Convert the problem into a Markov chain and give solution.
(b) For the two-state Markov chain $P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}, 0 < a, b < 1$.Prove that $\lim_{n\to\infty} p_{i1} = \frac{b}{a+b}$ and $\lim_{n\to\infty} p_{i2} = \frac{a}{a+b}$, i = 1,2.

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