THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 4434 **Marks: 75**
REAL ANALYSIS-II **Time: 3 Hrs**

Answer any FIVE questions **5x15=75**

1. Assume that α increases monotonically and $\alpha' \in \mathcal{R}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Then $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$ and $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.

2. If γ' is continuous on $[a,b]$, then γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$.

3. Suppose $\{f_n\}$ is a sequence of functions differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,b]$. If $\{f_n'\}$ converges uniformly on $[a,b]$ then $\{f_n\}$ converges uniformly on $[a,b]$ to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x)$ $(a \le x \le b)$.

4. If f is a continuous complex function on $[a,b]$, then prove that there exists a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on $[a,b]$.

5. State and prove Parseval's theorem.

6. (a) Let $\{\phi_n\}$ be orthonormal on $[a,b]$. Let $s_n(x) = \sum_{m=1}^{n} c_m\phi_m(x)$ be the n^{th} partial sum of the Fourier series of f and suppose $t_n(x) = \sum_{m=1}^{n} \gamma_m\phi_m(x)$ then $\int_a^b |f - s_n|^2 dx \le \int_a^b |f - t_n|^2 dx$ and the equality holds iff $\gamma_m = c_m$.
   (b) If X is a complete metric space, and if φ is a contraction of X into X, then there exists one and only one $x \in X$ such that $\varphi(x) = x$.

7. State and prove inverse function theorem.

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