THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT/MAS 2516 **Time: 3 hours**

VECTOR CALCULUS & TRIGONOMETRY **Max marks: 75**

PART-A

Answer any FIVE Questions: $(\mathbf{5 \times 15 = 75}$**marks)**

1. (i) Show that $\left|\overrightarrow{a \times b}\right|^2 + \left|\overrightarrow{a.b}\right|^2 = |\vec{a}|^2\left|\vec{b}\right|^2$

   (ii) Show that the volume V of a pyramid of which the vertex is at a given point (x, y, z) and the base of a triangle formed by joining three given points $(a, 0,0)$, $(0, b, 0)$, $(0,0, c)$ in the rectangular coordinate axes is $V = \frac{1}{6}\,\text{abc}\left(\frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1\right)$.

2. (i) $\vec{v} = \vec{w} \times \vec{r}$, prove that $\vec{w} = \frac{1}{2} curl\ \vec{v}$, where $\vec{w}$ is a constant vector and $\vec{r}$ is the position vector.

   (ii) Determine $f(r)$ so that the vector $\{f(r)\vec{r}\}$ is both solenoidal and irroational.

3. (i) Prove that $\boldsymbol{F} = (y^2 \cos x + z^3)\vec{\imath} + (2y \sin x - 4)\vec{\jmath} + (3xz^2 + 2)\vec{k}$ is irrotational and find its scalar potential.

   (ii) Prove that $\boldsymbol{v} = r^n\boldsymbol{r}$ irrotational. Find n when it is also solenoidal.

4. The necessary and sufficient condition that the line integral $\int_A^B \boldsymbol{F}.\boldsymbol{dr}$ be independent of the path is that **F** is the gradient of some scalar function φ.

5. (i) Evaluate $\iiint \vec{F}.dV$, where $\vec{F} = 2xz\,\vec{\imath} - x\,\vec{\jmath} + y^2\vec{k}$ and V is the volume enclosed by the cylinder $x^2 + y^2 = a^2$ between the plane $z = 0$ and $z = c$.

   (ii) Verify the Divergence theorem for the function $\vec{F} = 2xz\,\vec{\imath} + yz\,\vec{\jmath} + z^2\vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

6. (i) Solve the equation $\sin 7\theta - \sin\theta = \sin 3\theta$.

(ii) Express $\cos 8\theta$ in terms of $\sin\theta$.

7. (i) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + p = 0$, prove that $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = n\pi$ radians except $q = 1$.

   (ii) Prove that the equation $^{ah}/_{cos\theta} - {}^{bk}/_{sin\theta} = a^2 - b^2$ has four roots and that the sum of the four values of θ which satisfy it is equal to an odd multiple of π radians.
