



THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT/MAS 1412

Analysis - I

MAX: 75 marks

TIME: 3 hours

Answer Any FIVE of the following questions

5 × 15 = 75

- State and prove Archimedean property of \mathbb{R} .
 - Prove that countable union of countable set is countable.
- If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $c \in \mathbb{R}$, and if $\lim_{n \rightarrow \infty} S_n = L$ and $\lim_{n \rightarrow \infty} t_n = M$ then prove that
 - $\lim_{n \rightarrow \infty} (S_n + t_n) = L + M$
 - $\lim_{n \rightarrow \infty} c S_n = cL$
- If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers then prove that $\liminf_{n \rightarrow \infty} S_n \leq \limsup_{n \rightarrow \infty} S_n$.
 - Prove that any convergent sequence is a Cauchy sequence.
- State and prove Nested interval theorem.
- If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that (i) $a_1 \geq a_2 \geq \dots \geq a_n \geq a_{n+1} \dots$ (ii) $\lim_{n \rightarrow \infty} a_n = 0$ then prove that $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent.
- State and prove comparison test for series .
- If $\sum_{n=1}^{\infty} a_n$ converges absolutely then prove that $\sum_{n=1}^{\infty} a_n$ converges.
 - Test the convergence of the series $\sum \frac{2^n n!}{n^n}$.