THE AMERICAN COLLEGE, MADURAI
(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade 'A',CGPA-3.46 on a 4-point scale
(Satellite Campus, Chatrapatti)
Department of Mathematics

SINCE 1881

Max : 75 Marks

MAT 3544   **Stochastic Process**   **Time : 3 hours**

$(5 \times = 15 = 75)$

Answer any FIVE questions :-

1. Sole the equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = \sin x$ given that $y = \frac{dy}{dx} = 0$ $when$ $x = 0$ using Laplace transform.

2. Consider the random process $X(t) = \cos(\omega t + \theta)$, $where$ θ is uniformly distributed in the interval $-\pi$ to π. Check whether $X(t)$ is stationary or not?

3. The three state Markov chain is given by the transition probability matrix
$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$
Find the steady state distribution of the chain.

4. Define Poisson process and find its mean and variance.

5. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute ; find the probability that during a time interval of 2 minutes
(i) exactly 4 customers arrive
(ii) more than 4 customers arrive
(iii) fewer than 4 customers in 2 minutes interval.

6. Write a note on Birth and Death process in queuing theory.

7. Consider a queueing system where customers arrive according to a Poisson process with a rate of 25 customers per hour. The customers have exponential service requirements. We have a choice of using either two servers, each processing customers in 4 minutes on average,or a single server processing customers in 2 minutes on average.

What is

a) the expected time in the queue?

b) the probability that the time in the queue is greater than 12 minutes?

c) the expected time in the system?

d) the probability that the time in the system is greater than 12 minutes?

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