

## THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University) Re-accredited (2<sup>nd</sup> Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

## Backlog Arrear Examination, March 2021

| PGM 4332              | Time: 3Hours  |
|-----------------------|---------------|
| DIFFERENTIAL GEOMETRY | Max: 75 Marks |
|                       |               |

## ANSWER ANY FIVE QUESTIONS.

<u>5X15=75</u>

- 1. State and prove Serret-Frenet equations.
- 2. Prove that the position vector of any point on the surface of revolution generated by the curve [g(u), 0, f(u)] in the *XOZ* plane is  $r = [g(u) \cos v, g(u) \sin v, f(u)$  where *v* is the angle of rotation about the *z*-axis.
- 3. Show that any curve u = u(t), v = v(t) on a surface r = r(u, v) is a geodesic if and only if the principal normal at every point on the curve is normal to the surface.
- 4. If  $\kappa_n$  is the normal curvature of a curve at a point on a surface, then  $\kappa_n = \frac{Ldu^2 + 2Mdudv + Ndv^2}{Edu^2 + 2Fdudv + Gdv^2}$ , where  $L = N \cdot r_{11}$ ,  $M = N \cdot r_{12}$ ,  $N = N \cdot r_{22}$  and E, F, G are first fundamental coefficients.
- 5. Show that the only compact surfaces of class  $\geq 2$  for which every point is an umbilic are spheres.
- 6. State and prove existence theorem for space curves.
- 7. Prove that: a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.