



THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade “A”, CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

Course Code : MAT/MAS 1512

Course Title : Algebra – I

Time : 3 Hrs

Max : 75 Marks

$5 \times 15 = 75$

Answer Any FIVE questions :

- Show that $((P \vee Q) \wedge \sim (\sim P \wedge (\sim Q \vee \sim R))) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$ is a tautology.
 - Show that $(\exists x)M(x)$ follows logically from the premises $(\forall x)H(x) \rightarrow M(x)$ and $(\exists x)H(x)$.
- If ρ and σ are equivalence relations defined on a set S, prove that $\rho \cap \sigma$ is an equivalence relation
 - Show that $f: R - \{3\} \rightarrow R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is a bijection and find its inverse.
- Prove that the union of two subgroups of a group G is a subgroup iff one is contained in the other.
 - Let $f: A \rightarrow A$ be any function. Then $f \circ i_A = i_A \circ f = f$.
- State and prove Lagrange's theorem with necessary lemma.
- Let N be a normal subgroup of a group G. Then G/N is a group under the operation defined by $NaNb = Nab$.
- Any finite cyclic group of order n is isomorphic to (Z_n, \oplus) .
 - If $f: G \rightarrow G'$ be an isomorphism then prove that
 - $f(e) = e'$
 - $f(a^{-1}) = [f(a)]^{-1}$
- State and prove fundamental theorem of homomorphism.