THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT 2632 **LINEAR ALGEBRA AND LATTICES** **75 Marks**

Answer any FIVE Questions **5 X 15 = 75**

1. **(a)** Let V be a vector space over a field F and S be a non-empty subset of V. Then prove that
(i) S ⊆ T ⇒ L(S) ⊆ L(T)
(ii) L(S∪T) = L(S) + L(T)
(iii) L(S) = S iff S is a subspace of V

(b) Let V be a vector space over a field F. Let S = { $v_1, v_2, \dots \dots \dots v_n$ } and L(S) = W. Prove that there exists a linearly independent subset S' of S such that L(S') = W

2. **(a)** Prove that S = { $v_1, v_2, \dots \dots \dots v_n$ } is a linearly dependent vectors iff there exists a vector v_k such that it can be written as a linear combination of the preceding vectors $v_1, v_2, \dots \dots \dots v_{k-1}$.

(b) Let V be a vector space over a field F, S = { $v_1, v_2, \dots \dots \dots v_n$ } span V. Let $S' =$ { $w_1, w_2, \dots \dots \dots w_n$} be a linearly independent, prove that m ≤ n.

3. (a) Let V be a finite dimensional vector space over a field F. Let S = { $v_1, v_2, \dots \dots \dots v_n$ }⊆ V . Prove that S is a basis iff S is a maximal linearly independent set.
(b) Let V be a finite dimensional vector space over a field F. Let A and B be subspaces of the vector space V. Then prove that dim (A+B) = dim (A) + dim (B) – dim (A∩B)

4. **(a)** Prove that every finite dimensional inner product space as an orthonormal basis.
(b) Let $W_1$and W_2 be subspaces of a finite dimensional inner product space. Prove that
(i) $(W_1 \cap W_2)^{\perp} = {W_1}^{\perp} + {W_2}^{\perp}$
(ii) $(W_1 + W_2)^{\perp} = {W_1}^{\perp} \cap {W_2}^{\perp}$

5. Starting with the basis {1, x, x^2 } obtain the orthonormal basis for the Vector space of all polynomials of degree ≤ 2 together with zero polynomial and inner product given by
$\langle f, g \rangle = \int_{-1}^{1} f(x)\, g(x) dx.$

6. Prove that an isomorphism from a finite dimensional vector space takes a basis onto a basis.

7. (a) Let L be a non-empty set with two binary operations ∧ and ∨ such that it satisfies idempotent, commutative, associative and absorption laws. Prove that it is a lattice under suitable order ≤ in L where a ∧ b = glb(a,b) and a ∨ b = lub(a,b).
(b) Prove that in a lattice L, L_5 : a ∨ (b ∧ c) = (a ∨ b) ∧ (a ∨ c) and L_5 : a ∧ (b ∨ c) = (a ∧ b) ∨ (a ∧ c) are equivalent.