THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT 152 **Max: 75 marks**

Foundation Mathematics III **Time: 3 hrs**

Answer any five questions: $5 \times 15 = 75$ marks

1. a) Derive a reduction formula for $\int \sin^m x \cos^n x \, dx$ where m, n being positive integers.

   b) Evaluate $\int x^3 \cos 2x \, dx$.

2. a) By changing the order of integration, evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dx dy$.

   b) Find the area of the surface of the sphere of radius r.

3. a) Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

   b) Evaluate $\int_0^\infty e^{-x^2} dx$.

4. Show that the straight lines whose direction cosines are given by $al + bm + cn = 0$, $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and parallel if $\sqrt{af} + \sqrt{bg} + \sqrt{ch} = 0$.

5. a) Show that the origin lies in the acute angle between the polanes x+2y+2z=9, 4x-3y+12z+13=0. Find the planes bisecting the angles between them and point out which bisects the obtuse angle.

   b) Find the shortest distance between the lines

   $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{(z+2)}{1}; \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$.

6. a) Determine $f(r)$ so that the vector $\{f(r)\boldsymbol{r}\}$ is both solenoidal and irrotational.

   b) Prove that $\boldsymbol{F} = (y^2 cosx + z^3)\boldsymbol{i} + (2ysinx - 4)\boldsymbol{j} + (3xz^2 + 2)\boldsymbol{k}$ is irrotational and find its scalar potential.

7. Verify the Gauss divergence theorem for the function $\boldsymbol{F} = 2xz\boldsymbol{i} + yz\boldsymbol{j} + z^2\boldsymbol{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.