THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Since 1881

Backlog Arrear Examination, March 2021

MAT/MAS 2514 **Analysis - III** **MAX: 75 marks**

TIME: 3 hours

Answer Any FIVE of the following questions **5 ×15 = 75**

1. Let f be a bounded function on the closed bounded interval $[a,b]$.
   Prove that $f \in \Re[a,b]$ iff f is continuous at almost every point in $[a,b]$

2. If $f \in \Re[a,b]$, $g \in \Re[a,b]$ then prove that $f+g \in \Re[a,b]$ and $\int_a^b f+g = \int_a^b f + \int_a^b g$.

3. If f is continuous function on the closed bounded interval [a, b] and if $f'(x)$ exists for all $x \in (a,b)$ prove that there exists $c \in (a,b)$such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

4. State and prove Taylor's formula with Integral and Lagrange form of the remainder.

5. Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of real valued function on a set E. Prove that $\{f_n\}_{n=1}^{\infty}$ is Uniformly convergent to E iff given $\varepsilon > 0$, there exists N$\in I$ suchthat $|f_m(x) - f_n(x)| < \varepsilon, m,n \geq N, x \in I$.

6. State and prove Dini's theorem for sequence.

7. (i) state and prove Weierstrass M-test.
   (ii) Find the radius of convergence for the series $\sum \frac{n+1}{(n+1)(n+2)}$.