THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT/MAS 2444/2604/234/254/2634 Real Analysis - II MAX: 75 marks

TIME: 3 hours

Answer Any FIVE of the following questions **5 ×15** = 75

1. Let (M, d) be a metric space. Let A, B⊆ M. Prove that
   (i) A is open iff A = Int A
   (ii) Int A = Union of all open sets contained in A
   (iii) Int A is an open subset of A and if B is any other open set contained in A then B ⊆ Int A.
   (iv) A⊆ B ⟹Int A⊆ Int B
   (v) Int$(A \cap B)$ = IntA ∩ Int B
   (vi) Int$(A \cup B) \supseteq$ IntA ∪ Int B

2. Let M be a metric space and A⊆M . Prove that $\bar{A} = A \cup D(A)$.

3. State and prove Heine Borel theorem.

4. Let f be a bounded function on the closed bounded interval $[a, b]$. Prove that $f \in \Re[a, b]$iff$f$ is continuous at almost every point in $[a, b]$

5. State and prove Dini's theorem for sequence.

6. State and prove Weierstrass M – test .

7. Obtain the Fourier series for f$(x) = x - x^2$ in $-\pi < x < \pi$. Hence prove that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$.