THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 5542 /5432/5442 **MAX: 75 MARKS**

FUNCTIONAL ANALYSIS **TIME: 3 HOURS**

ANSWER ANY 5 QUESTIONS **(5 x 15 =75)**

1) Prove that the following are equivalent in a normed linear space X.
   i. Every closed and bounded subset of X is compact.
   ii. The closed unit ball in X is compact.
   iii. X is finite dimensional.

2) Let X be a normed linear spaces over K, E be a non empty convex open subset of X and Y be a sub space of X such that $E \cap Y = \phi$.Prove that there is a closed hyper space Z of X such that $Y \subset Z$ and $E \cap Z = \phi$.Consequently there is some . $f \in X'$ such that $f(x) = 0$ for all $x \in Y$ but $\operatorname{Re} f(x) \neq 0$ for all x in E.

3) a) Prove that a normed space is Banach iff every absolutely summable series in it is summable

   b) Prove that a Banach space cannot have a denumerable basis.

4) a)State and prove uniform boundedness principle.
   b) Let X be a Banach space and Y be a normed space , $F_n \in BL(X,Y)$ such that $(F_n(x))$ converges in Y for every x in X
   Define . $F(x) = Lim_{n\to\infty} F_n(x)$ for each x in X.
   Prove that if E is totally bounded in X, the convergence above is uniform.

5) State Riesz representation theorem.

6) Let $\{x_n\}$ be a sequence in a Hilbert space H. Then prove the following.
   a) $x_n \to x$ iff $x_n \xrightarrow{w} x$ and $LimSup_{n\to\infty} \|x_n\| \leq \|x\|$.
   b) If $\{x_n\}$ is bounded, then it has a weak convergent subsequence.

7) Prove that a subset of a Hilbert space is weak bounded iff it is bonded.