THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 4432 **ALGEBRA-II** **75 Marks**

Answer any FIVE Questions **5 X 15 = 75**

1. If L is a finite extension of F and if K is a subfield of L which contains F, then prove that [K : F] | [L : F] .
2. If p(x) is a polynomial in F[x] of degree $n \geq 1$ and is irreducible over F, then prove that there is an extension E of F, such that [E : F] = n, in which p(x) has a root.
3. Prove that K is a normal extension of F iff K is the splitting field of some polynomial over F.
4. State and prove fundamental theorem of Galois theory.
5. Let V be an n – dimensional vector space over the field F, and let W be an m – dimensional vector space over F. Then prove that the space L(V, W) is finite dimensional and has dimension mn.
6. Let T be a linear operator on a finite dimensional space V. Let $c_1, \ldots, c_k$ be the distinct characteristic values of T and let W_i be the null space of $(T - c_iI)$. Prove that the following are equivalent:

   (i) T is diagonalizable.

   (ii) The characteristic polynomial for T is $f = (x - c_1)^{d1} \ldots (x - c_k)^{dk}$ and $\dim W_i = d_i$, $i = 1, \ldots, k$.

   (iii) $\dim W_1 + \ldots + \dim W_k = \dim V$.
7. State and prove Primary Decomposition theorem.