THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT/MAS 2513 **Analysis - II** **MAX: 75 marks**

TIME: 3 hours

Answer Any FIVE of the following questions **5 ×15 = 75**

1. (i) Show that $f(x) = |x| + |x-1|$ is continuous at $x = 0$ and $x = 1$.
   (ii) State and prove Intermediate value theorem.
2. Let (M, d) be a metric space. Let A, B⊆ M. Prove that
   (i) A is open iff A = Int A
   (ii) Int A = Union of all open sets contained in A
   (iii) Int A is an open subset of A and if B is any other open set contained in A then B ⊆ Int A.
   (iv) A⊆ B ⟹Int A⊆ Int B
   (v) Int$(A \cap B)$ = IntA ∩ Int B
   (vi) Int$(A \cup B) \supseteq$ IntA ∪ Int B

3. Let M be a metric space and A⊆M . Prove that $\bar{A} = A \cup D(A)$.

4. Prove that a subspace of R is connected iff it is an interval.

5. State and prove Cantor's Intersection theorem.

6. Prove that any complete metric space is of second category.

7. Prove that any closed interval [a, b] is a compact subset of R.