



THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 5433 MEASURE THEORY

TIME: 3 hrs

Max.Marks: 75

Answer any FIVE questions

$5 \times 15 = 75$

1. For any set A , prove that there exists a measurable set E containing A such that

$$m^*(A) = m^*(E)$$

2. Prove that the existence of non measureable set.

3. If $\{f_n\}_{n=1}^{\infty}$ is a sequence of non negative measurable functions such that $\{f_n(x)\}_{n=1}^{\infty}$ for each 'x' and $f = \lim f_n$, then prove that $\int f dx = \lim \int f_n dx$

4.) If f is Riemann integrable and bounded over the finite interval $[a, b]$, then prove

$$\text{that } f \text{ is integrable and } R \int_a^b f dx = \int_a^b f dx$$

5. Prove that the space $C[a, b]$ is dense in $L_1[a, b]$

6. Show that S^* forms a σ -Ring

7. State and prove Hahn Decomposition Theorem (Prove the necessary result). Is the decomposition unique? Justify