THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 4433 /PGM 453   **Marks: 75**
REAL ANALYSIS-I   **Time: 3 Hrs**

Answer any FIVE questions   **5x15=75**

1. If a and b are positive integers and n is positive integer then prove that $(ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}}$.

2. If F is closed and K is compact subsets of a compact metric space (X, d) then prove that $F \cap K$ is compact.

3. Prove that every $k-$cell is compact.

4. Prove that $\lim\limits_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$. Also prove that e is irrational.

5. Suppose
   a) $|c_1| \geq |c_2| \geq |c_3| \geq \cdots$.
   b) $c_{2m-1} \geq 0, c_{2m} \leq 0 \quad (m = 1,2,3, \dots.)$
   c) $\lim\limits_{n\to\infty} c_n = 0$.
   Then prove that $\sum c_n$ converges.

6. a) Prove that a mapping f of metric space X into a metric space is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.
   b) Prove that continuous image of a compact set is compact.

7. State and prove L'Hospital's rule.

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