



THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 5437
COMPLEX ANALYSIS

Time: 3Hours
Max: 75 Marks

ANSWER ANY FIVE QUESTIONS.

5X15=75

1. Prove that: if $\sum a_n(z - a)^n$ is a given power series with radius of convergence R , then $R = \lim \left| \frac{a_n}{a_{n+1}} \right|$ if this limit exists.
2. Let u and v be real valued functions defined on a region G and suppose that u and v have continuous partial derivatives. Then prove that: $f: G \rightarrow \mathbb{C}$ defined by $f(z) = u(z) + iv(z)$ is analytic if and only if u and v satisfy the Cauchy-Riemann equations.
3. If $\gamma: [a, b] \rightarrow \mathbb{C}$ is piecewise smooth, then show that γ is of bounded variation and
$$V(\gamma) = \int_a^b |\gamma'(t)| dt$$
4. State and prove Goursat's theorem.
5. State and prove open mapping theorem.
6. State and prove Cauchy's Residue theorem.
7. State and prove Hadamard's Three Circles Theorem.