# THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

## Backlog Arrear Examination, March 2021

**MAT/MAS 3543/3612 Number Theory** **MAX: 75 marks**

**TIME: 3 hours**

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**Answer Any FIVE of the following questions** **5 ×15 = 75**

1. State and prove fundamental theorem of Arithmetic.

2. Prove that the linear diophantine equation $ax + by = c$ has a solution iff $d|c$where $d = gcd(a, b)$. Also if $x_0$,$y_0$ is any particular solution of this equation then prove that all other solutions are given by $x = x_0 + \left(\frac{b}{d}\right)t$; $y = y_0 - \left(\frac{a}{d}\right)t$ for varying integer t.

3. State and prove chinese Remainder theorem. Also solve the following set of simultaneous congruence
   $x \equiv 2(mod\ 3)$ ; $x \equiv 3(mod\ 5)$; $x \equiv 2(mod\ 7)$ .

4. (i) If $p$ is a prime then prove that $(p-1)! \equiv -1(mod\ p)$.
   (ii) Show that $18! \equiv -1(mod\ 437)$.

5. State and prove Gauss Lemma .Also find the Legendre symbol $(19/23)$and $(8/11)$ .

6. If $p$ and $q$ are distinct odd primes then prove that $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2}\cdot\frac{q-1}{2}}$

7. (i) If $2^k - 1$ is a prime $(k > 1)$,prove that $n = 2^{k-1}(2^k - 1)$ is perfect and every even perfect number is of this form.
   (ii) State and prove Mobius Inversion formula.