THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT/MAS 3511/3541 – COMBINATORICS

Max : 75 Mark **Time : 3 Hours**

Answer Any FIVE Questions (5 × 15 = 75 Marks)

1. How many ways can a committee is to be formed from four men and seven women with
   i) The committee can be any positive size but must have equal number of men and women.
   ii) The committee has 4 people, two of which are women, and Mr. and Mrs. X cannot both be on the committee.
   iii) The committee has 5 people, 3 women and 2 men.
2. There are 4 different roads from town A to town B, 3 different roads from town B to town C, and 2 different roads from town A to town C.
   i) How many different routes from A to C altogether?
   ii) How many different routes from A to C and Back (any road can be used once in each direction)?
   iii) How many different routes from A to C and back that visit B at least once?
   iv) How many different routes from A to C and back that do not use any road twice?
3. (i) Using Exponential generating function, find the number of ways to place 25 people into three rooms with atleast one person in each room.
   (ii) Build a generating function $h(x)$ with $a_r = (r+1)r(r-1)$
4. Solve the recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$ with $a_0 = a_1 = 1$
5. An elf has a staircase of n stairs to climb. Each step it takes can cover either one stair or two stairs. Find a recurrence relation for a_n, the number of different ways for the elf to ascend the $n-$stair staircase. Also solve it
6. (i) Define Stirling number of first kind and signess stirling number of first kind.
   (ii) Prove that $S'(n+1,r) = S'(n,r-1) + nS'(n,r)$.
   Hence construct the table up to rows
7. (i) How many integers less than 280 are relatively prime to 280?
   (ii) How many integer solutions are there to the equation
   $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 20$ with $0 \le x_i \le 8$