



# THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)  
Re-accredited (2<sup>nd</sup> Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

## Backlog Arrear Examination, March 2021

MAT/ MAS 1441 / 1631

Calculus

Marks: 75

Time: 3 Hrs

Answer any FIVE questions:

(5 X 15 = 75 Marks)

1. Find the evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

2. Discuss the maxima and minima of the function  $x^3y^2(6-x-y)$ .

3. If  $\int_0^{\frac{\pi}{2}} \cos^m x \cos nx dx = f(m, n)$  then prove that  $f(m, n) = \frac{m}{m+n} f(m-1, n-1)$ .

Hence prove that  $f(n, n) = \frac{\pi}{2^{n+1}}$ .

4. Prove that  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$  and hence evaluate  $\Gamma(1/2)$ .

5. Evaluate  $\iiint_D xyz dx dy dz$  where D is the positive octant of the sphere  $x^2 + y^2 + z^2 = 1$ .

6. By transforming into polar co-ordinates evaluate  $\iint \frac{x^2y^2}{x^2+y^2} dx dy$  over the region between the circles  $x^2 + y^2 = a^2$  and  $x^2 + y^2 = b^2$  ( $b > a$ ).

7. (i) Prove that the radius of curvature at any point of the catenary  $y = c \cosh \frac{x}{c}$  is  $\frac{y^2}{c}$ .

(ii) If  $U = \tan^{-1} \frac{x^3 + y^3}{x - y}$ , prove that  $x \frac{\partial U}{\partial x} + y \frac{\partial U}{\partial y} = \sin 2U$ .

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