



THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT/MAS 1413

Time: 3 hours

DIFFERENTIAL CALCULUS

Max marks: 75

Part-A

Answer any FIVE Questions

(5 × 15 = 75 marks)

- (i) Differentiate $\frac{\sin x}{1+\tan x}$ with respect to x .

(ii) If $x = a(t - \sin t)$ and $y = a(1 - \cos t)$ find $\frac{dy}{dx}$.
- (i) Find y_n where $y = \frac{x^2}{(x-1)^2(x+2)}$

(ii) If $y = (x + \sqrt{1+x^2})^m$ show that
 $(x^2 + 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.
- (i) Find the maximum value of $\frac{\log x}{x}$ for positive values of x .

(ii) Find the condition that the curves $ax^2 + by^2 = 1$, $a_1x^2 + b_1y^2 = 1$ shall cut orthogonally.
- The greatest value of $ax + by$ where x and y are positive and $x^2 + xy + y^2 = 3k^2$ is $2k\sqrt{a^2 - ab + b^2}$.
- Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
- A tent having the form of a cylinder surmounted by a cone is to contain a given volume. If the canvass required is minimum, show that the altitude of the cone is twice that of the cylinder.
- If $u = a^3x^2 + b^3y^2 + c^3z^2$ where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find the minimum value of u .