



THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT/MAS 2443/2603/233/2633 Real Analysis - I

MAX: 75 marks

TIME: 3 hours

Answer Any FIVE of the following questions

5 × 15 = 75

1. If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers prove that $\liminf_{n \rightarrow \infty} S_n \leq \limsup_{n \rightarrow \infty} S_n$.
2. State and prove comparison test for series of positive terms.
3. Let M be a metric space. Prove that M is complete iff for every sequence (F_n) of non-empty closed subsets of M such that $F_1 \supseteq F_2 \supseteq \dots \supseteq F_n \supseteq \dots$ and $(d(F_n)) \rightarrow 0$ and $\bigcap_{n=1}^{\infty} F_n$ is non empty.
4. State and prove Nested interval theorem.
5. (i) Show that $f(x) = |x| + |x - 1|$ is continuous at $x = 0$ and $x = 1$.
(ii) If a function f is continuous in an interval $[a, b]$ and $f(a) \neq f(b)$, then prove that f assumes every value between $f(a)$ and $f(b)$.
6. Prove that any closed interval $[a, b]$ is a compact subset of R.
7. (i) State and prove Rolle's theorem
(ii) State and prove Mean value theorem.