THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 5545/5435 **Time: 3 Hrs**

STATISTICS **Marks: 75**

Answer any **FIVE** questions. **5 x 15 = 75**

1) State and prove Chebyshev's inequality.

2) Let (X_1, X_2) be a random vector such that the variance of X_2 is finite. Then prove that

   (a) $E\left[E\left(\frac{X_2}{X_1}\right)\right] = E(X_2)$

   (b) $var\left[E\left(\frac{X_2}{X_1}\right)\right] \leq var\,(X_2)$

3) State and prove Student's Theorem.

4) State and prove Central Limit Theorem.

5) Prove the following

   (i) Suppose $X_n \xrightarrow{p} X$ and $Y_n \xrightarrow{p} Y$. Then prove that $X_n + Y_n \xrightarrow{p} X + Y$.

   (ii) Suppose $X_n \xrightarrow{p} X$ and a is a constant. Then prove that $X_n \xrightarrow{p} aX$.

   (iii) Suppose $X_n \xrightarrow{p} a$ and the real function g is continuous at a. Then prove that

   $$g(X_n) \xrightarrow{p} g(a)$$

6) Discuss about confidence intervals for differences in means.

7) (i) Derive the mean and variance of Gamma distribution.

   (ii) Let X have a gamma distribution with $\alpha = {}^{r}/_{2}$, where $r$is a positive integer,and $\beta > 0$. Define the random variable $Y = 2X/\beta$. Then find the pdf of Y.

****