THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

PGM 5431/5441 **MAX:75 MARKS**
TOPOLOGY **TIME: 3 HOURS**

ANSWER ANY 5 QUESTIONS **(5 x 15 =75)**

1. i. Prove that R^w with product topology is metrizable.
   ii. Prove that R_l *and* R_K *are non comparable*
2. i. If the sets A and B form a separation for the topological space X, if Y is a connected subspace of X, then Y lies entirely within either A or B.
   ii. Prove that arbitrary union connected subspaces of a topological space X which have a point in common is also connected.
   iii. Hence prove that finite Cartesian product of connected spaces is connected.
   iv. Also prove that R^ω under product topology is connected.
3. i) State and prove Tube lemma
   ii) Hence prove that the product of finitely many compact spaces is compact.
   iii) Prove that continuous image of a compact set is compact
4. Prove : A topological space is locally compact Hausdorff space iff there exists a space Y such that
   (i) X is a subspace of Y.
   (ii) Y-X is a singleton set.
   (iii) Y is a compact Hausdorff space.
   Any other extension of X with the same properties must be homeomorphic.
5. State and prove Urysohn lemma
6. State and prove Tietze's extension theorem
7. State and prove Tychonoff Theorem

   OR

   State and prove Stone Cech Compactification Theorem