



THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)

Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT 153

Max: 75 marks

Foundation Mathematics II

Time: 3 hrs

Answer any five questions: $5 \times 15 = 75$ marks

- a) Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign.

b) If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two, prove that $p^3 + 8r = 4pq$.
- Use Newton's method of divisors to find all the rational roots of the equation

$$4x^3 + 20x^2 - 23x + 6 = 0$$
- a) If x, y, z be three consecutive integers, show that $(\sum x)^3 - 3\sum x^3$ is divisible by 108.

b) Find the highest power of 3 dividing 1000!.
- a) If p is a prime and a is any number prime to p then show that $a^{p-1} - 1$ is divisible by p .

b) Show that $(18)! + 1$ is divisible by 437.
- a) If x and y are positive quantities whose sum is 4, show that

$$\left(x + \frac{1}{x}\right)^2 + \left(y + \frac{1}{y}\right)^2 \geq 12\frac{1}{2}$$

b) Show that if a, b, c are positive unequal quantities then

$$ax^{b-c} + bx^{c-a} + cx^{a-b} > a + b + c .$$
- a) State and prove Weirstrass inequalities.

b) Find the maximum value of $(3 - x)^5(2 + x)^4$ when x lies between 3 and -2
- Sum the series $\frac{2}{1.4.5} + \frac{3}{2.5.6} + \frac{4}{3.6.7} + \dots$ to n terms.