THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT/MAS 2511 / 251 **TIME: 3 HRS**
ALGEBRA II **MAX: 75**

ANSWER ANY FIVE QUESTIONS **5× 15 = 75**

1. Prove that z_n is a field iff n is prime.(Prove necessary lemma)

2. Let R be a commutative ring with identity. Prove that an ideal of R is maximal iff R/M is a field.

3. Prove that any integral domain D can be embedded in a field F and every element of F can be expressed as a quotient of two elements of D.

4. Let R and R' be rings and $f: R \to R'$ be an isomorphism. Prove the following:

   (i) R is commutative ⇒ R' is commutative
   (ii) R is ring with identity ⇒ R' is a ring with identity
   (iii) R is an integral domain ⇒ R' is an integral domain
   (iv) R is a field ⇒ R' is a field.

5. Prove that any Euclidean domain R is a U.F.D (prove necessary lemma)

6. (a) State and prove Division algorithm.

7. (a) Show that in any distributive lattice $(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a)$.

   (b) Prove that the lattice of normal subgroup of any group is a modular lattice.

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