THE AMERICAN COLLEGE, MADURAI

(An Autonomous Institution Affiliated to Madurai Kamaraj University)
Re-accredited (2nd Cycle) by NAAC with Grade "A", CGPA – 3.46 on a 4-point scale

Backlog Arrear Examination, March 2021

MAT 331/351 **Time: 3Hours**
MECHANICS-I **Max: 75 Marks**

ANSWER ANY FIVE QUESTIONS. **5X15=75**

1. The resultant of two forces P, Q acting at a certain angle is X and that of P, R acting at the same angle is also X. The resultant of Q, R again acting at the same angle is Y. Prove that $P = (X^2 + QR)^{\frac{1}{2}} = \frac{QR(Q+R)}{Q^2+R^2-Y^2}$. Prove also that, if $P + Q + R = 0$ then $Y = X$.
2. ABC is a given triangle. Forces P, Q, R acting along the lines OA, OB, OC are in equilibrium. Prove that
   (a) $P: Q: R = a^2(b^2 + c^2 - a^2): b^2(c^2 + a^2 - b^2): c^2(a^2 + b^2 - c^2)$ if O is the circumcentre of the triangle.
   (b) $P: Q: R = \cos\frac{A}{2}: \cos\frac{B}{2}: \cos\frac{C}{2}$ if O is the incentre of the triangle.
3. A uniform rod, of length a, hangs against a smooth vertical wall being supported by means of a string, of length l, tied to one end of the rod, the other end of the string being attached to a point in the wall: Show that the rod can rest inclined to the wall at an angle θ given by $\cos^2\theta = \frac{l^2-a^2}{3a^2}$. What are the limits of the ratio $a: l$ in order that equilibrium may be possible?
4. A body is at rest on a rough inclined plane of inclination α to the horizon, being acted on by a force making an angle θ with the plane. Find the limit between which the force must lie and also find the magnitude and direction of the least force required to drag the body up the inclined plane.
5. Prove that the effect of a couple upon a rigid body is not altered if it is transferred to the parallel plane provided its moment remains unchanged in magnitude and direction.
6. ABCDEF is regular hexagon: forces P, 2P, 3P, 2P, 5P, 6P act along AB, BC, DC, ED, EF, AF respectively. Show that the six forces are equivalent to a couple and find the moment of the couple.
7. State and Prove Varigon's theorem.